RECAP DECREMENTAL SPANNER D.S.

Given a graph $G = (V, E)$ undergoing edge deletions & vertex splits,

(call these graphs $G = G_0, G_1, G_2, \dots, G_t$) ⌞ after t updates

→ We can maintain a spanner H_i of G_i (stretch $m^{o(1)}$) across the sequence of updates

$\tilde{O}(1) + \frac{t m^{o(1)}}{n}$

H_0 has $|E_{H_0}| \leq \tilde{O}(|V|)$

$t m^{o(1)}$

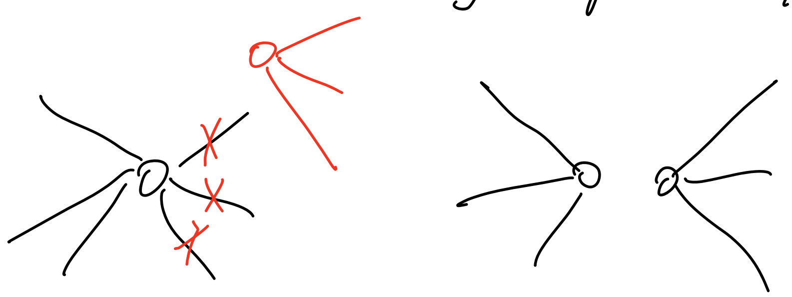
\# of changes from H_i to H_{i+1} is amortized $m^{o(1)}$.

Δ : max deg

$\leq n\Delta$ vertices

$\Delta = m^{o(1)}$

+ convert to fully dynamic.



USING FORESTS/CONTRACTION FOR VERTEX SPARSIFICATION

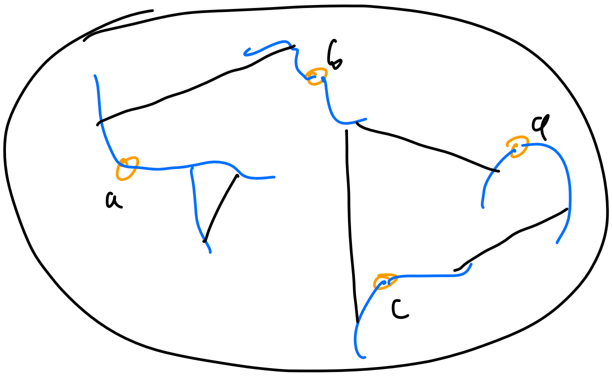
$G = (V, E)$

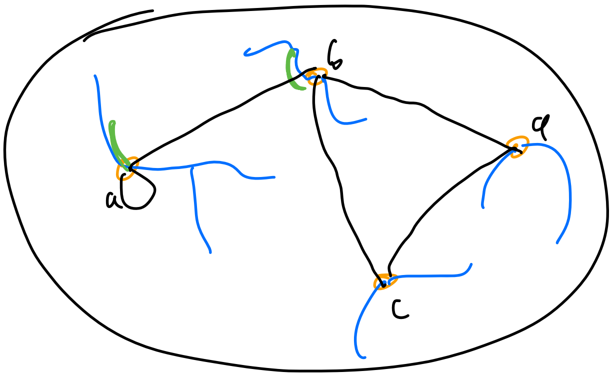
Rooted Forest F $\quad V_F = V$

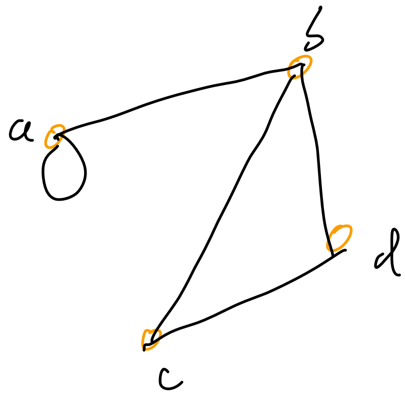
$E_F \subseteq E$

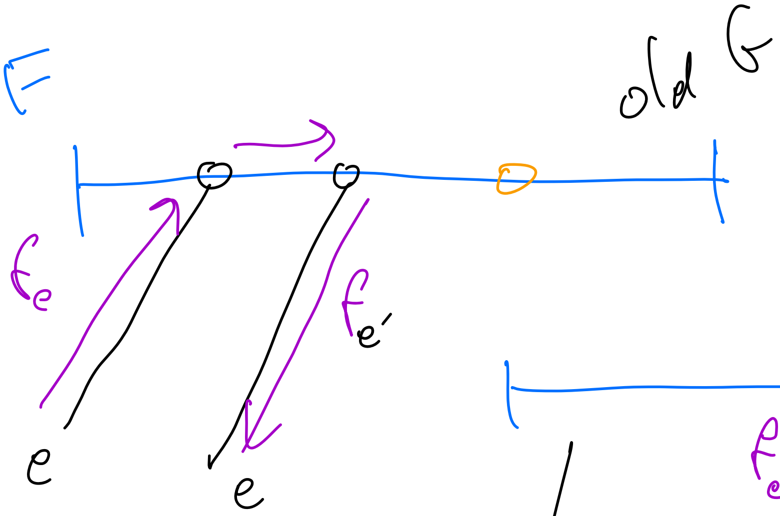
↓

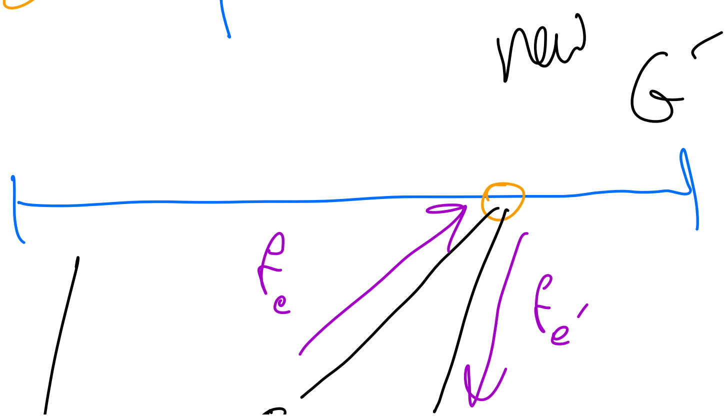
every connected component of F has a root vertex $r \in V$







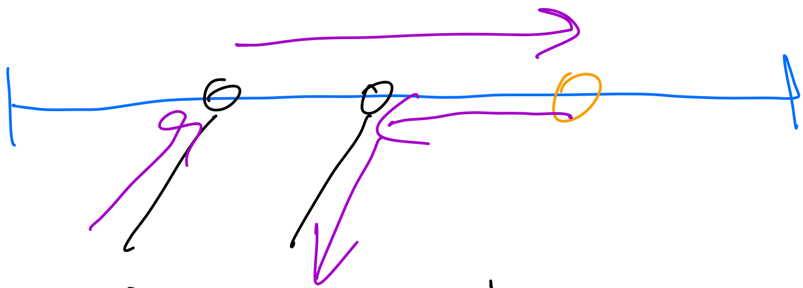




$\tilde{\ell}_{e'}$ $\quad f_{\ell_{e'}}$

map back

old graph



$$\ell_e' = \ell_e + O(\ell(e \text{ to root}))$$

$$\| L_{q^G} \Delta_{q^G} \| \qquad \Delta_G \to \Delta_{G'}$$

circulation

$$\| L_{G'}^{new} \Delta_{G'}^{new} \|_1 \le \tilde{O}(1) \| L_G^{old} \Delta_G^{old} \|_1$$

$$\| L_{old} \Pi_{new \to old} \Delta_{new} \|_1 \le \| L_{new} \Delta_{new} \|_1$$

map of flows from new graph G'

the old graph G

$$\| L_{G^{new}} \Delta_{G^{new}} \|$$

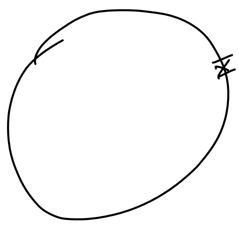
$$\leq \hat{O}(1) \| L_{G^{old}} \Delta_{G^{old}} \|$$

$$\hat{O}(1) \sum_e \ell_e |\Delta_e|$$

$$\geq \sum_e \left(\ell_e + \ell(e \text{ to root}) \right) |\Delta_e|$$

MWU Discussion

Räcke '08



n vertices
n edges

W worst case str

$\tilde{O}(1)$ avg stretch

average $\tilde{O}(W)$ trees

Madry '10

100m : at most $\frac{1}{10}$m

↓

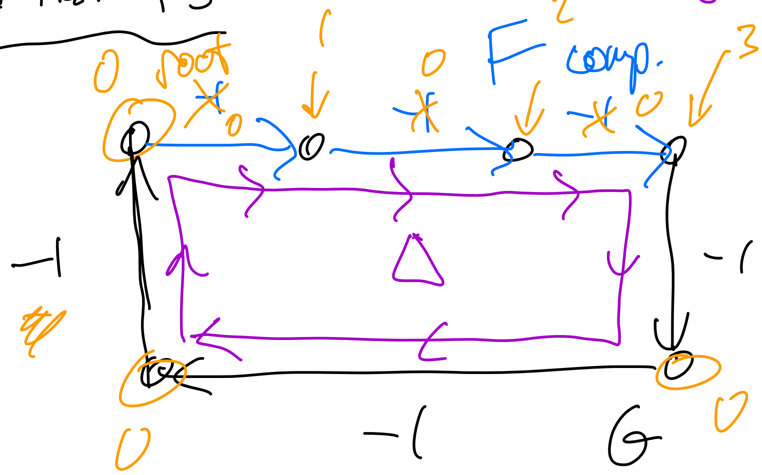
total stretch

stretch ≥ 1000

GRADIENTS

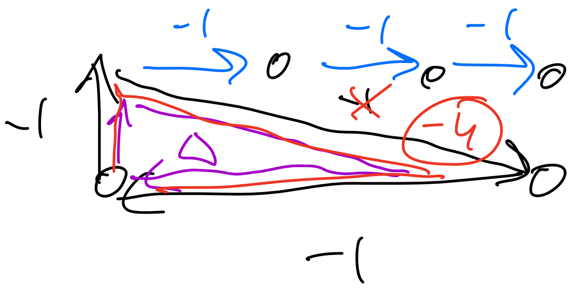
$g^T\Delta = -6$

cycle gradient



$g^T\Delta = -3$

cycle gradient



-6

$\langle g, \Delta \rangle$

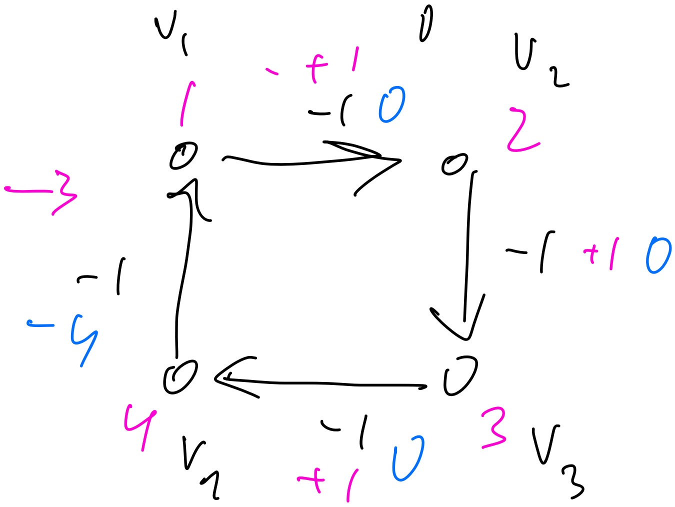
$\downarrow$

$\langle g + B^T \underline{v}, \Delta \rangle$

$B \Delta = 0$ — edge vertex incidence matrix

$$\underbrace{\boxed{B}}_{\text{vertices}}^{\text{edges}} \underbrace{\Delta}_{\text{flow}} = \underline{0}$$

$$\langle g, \Delta \rangle + \underbrace{\langle B^T v, \Delta \rangle}_{\langle v, \underbrace{B\Delta}_{0} \rangle}$$



$$v_2 - v_1$$
$$+ \; v_3 - v_2$$
$$+ \; v_4 - v_3$$
$$+ \; v_1 - v_4$$