DYNAMIC SPANNER

- 2) general intro to basic static spanner

- 1) general intro to expanders & expander decomp

0) THM

1) properties of our static spanner

2) use patching & batching of static spanner to build dynamic spanner

   A) lengths & recourse

   B) running time → vertex congestion

3) more about the static spanner

Stretch of an edge w.r.t. a subgraph

$G = (V, E)$, subgraph $H = (V, F)$ $F \subseteq E$

$\mathrm{str}_H(e) =$ Length of ~~shortest path in~~ embedding path in H
$e = (u, v)$ from u to v

Baswana-Sen '03

THM Graph $G = (V, E)$ n vertices, m edges

There exists a subgraph $H = (V, F)$, $F \subseteq E$

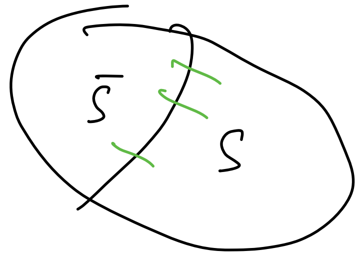
$|F| \leq \tilde{O}(n)$

with $\forall e \in E$ $\mathrm{str}_H(e) \leq \tilde{O}(1)$

- compute in time $\tilde{O}(m)$

DEFN $G=(V,E)$ is φ-expander iff

For all $S \subseteq V$



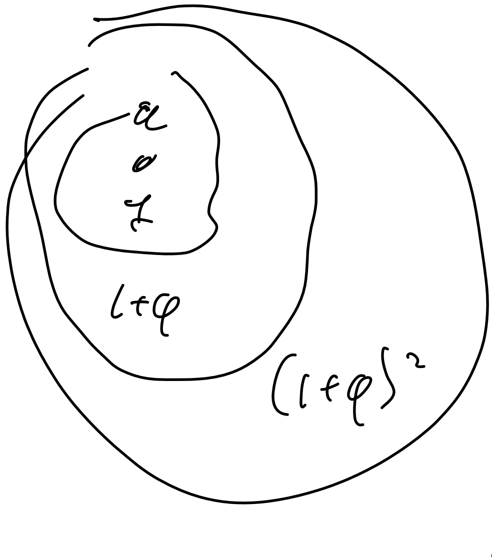
$$\frac{|E(S,\bar{S})|}{\min(vol(S), vol(\bar{S}))} \ge \varphi$$

$$vol(S) = \sum_{v \in S} deg(v)$$

CLAIM G φ-expander

$\Downarrow$

$$diam(G) \le \tilde{O}\left(\frac{1}{\varphi}\right)$$



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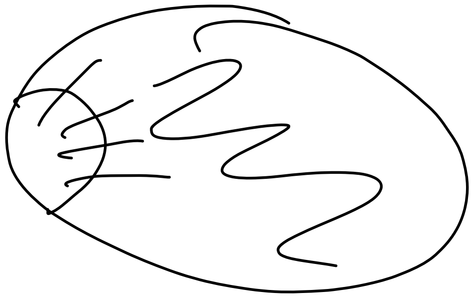


$(1+\varphi)^t$

$$t = \frac{\log m}{\varphi}$$

CLAIM Suppose G is a φ-expander w. min degree d

Make H by indep. sampling each edge of G w. prob $\tilde{O}\left(\frac{1}{d\cdot\varphi}\right)$

~~$\tilde{O}\left(\frac{1}{d}\right)$~~

Then whp. H is a $\varphi/2$ expander and has $\tilde{O}\left(\frac{n}{d\varphi}\right)$ edges.

Corollary: if the avg. deg. in G is $\tilde{O}(d)$ then H has $\tilde{O}\left(\frac{n}{\varphi}\right)$ edges

$$\frac{\frac{n\cdot d_{avg}}{2}}{d}$$

Corol. If $\frac{1}{\varphi} = \tilde{O}(1)$ as well then H is a decent sparser of G

Expander decomp Saranurak–Wang '2019

Given $G = (V, E)$ we can partition V into

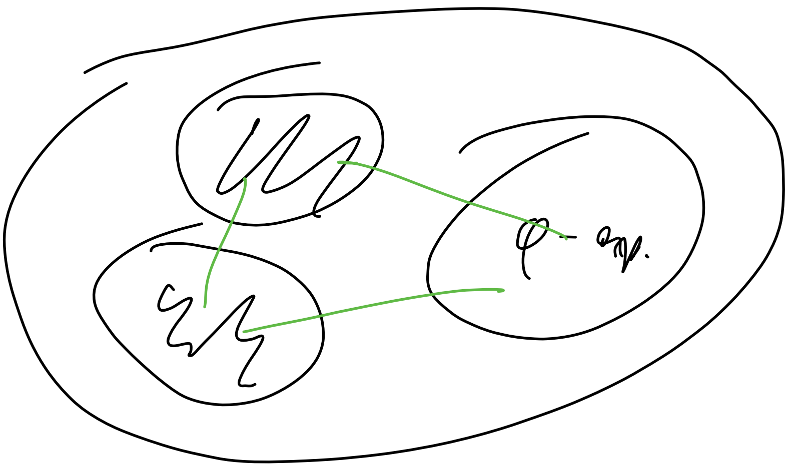
$V_1, \dots, V_k$

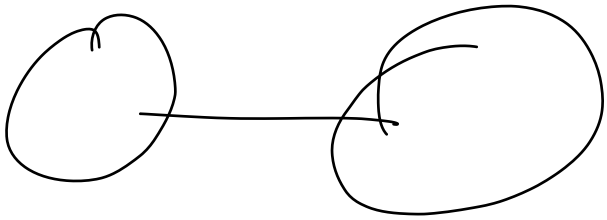
s.t. $G[V_j]$ is a φ-expander for all $j \in [k]$

$$\varphi = \frac{1}{\tilde{O}(1)} = \tilde{\Omega}(1)$$

with $\leq \frac{m}{\varphi}$ edges crossing the partition.

The algo runs in time $\tilde{O}(m)$.





Degree-controlled Exp D. THM

Suppose G has max deg Δ

then we can partition the edge set

into $1 + \log\Delta$ pieces $E_0, \dots, E_\ell$, $\ell = \log \Delta$

s.t. For $i > 0$ each conn. component of E_i

is $\tilde{\Omega}(1)$ expander

with min-deg $\underbrace{\Omega(2^i)}_{d_i}$

and $|E_i| \leq 2^i n$

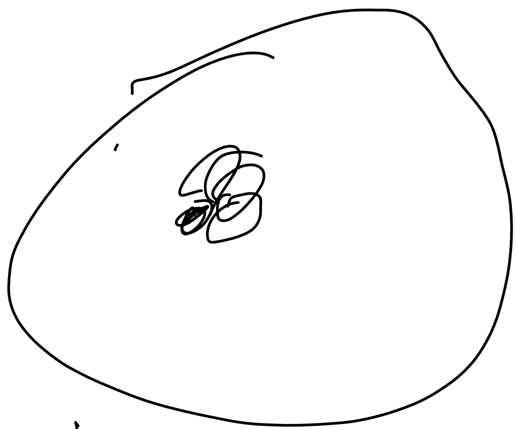
E_i

$|E_0| \leq n$

Pf sketch: $G_{\ell+1} \leftarrow G$

for $i = \ell, \dots, 1$

Form $G_i^{\circlearrowleft}$ add 2^i self loops per vertex to G_{i+1}



Apply E.D. to $G_i^{\circlearrowleft}$

C_i : crossing edges of E.D.

sets $V_1^{(i)}, \dots, V_k^{(i)}$ $\mathcal{E}^i$

$G_i \leftarrow (V, C_i)$

$|E(S, \bar{S})|$

edges internal to E.D.

$S \,/\!/\, v \in V_j^{(i)}$

$$\frac{\deg(v)}{\cancel{\mathrm{vol}(v)}} \geq \tilde{\Omega}(1)$$

$\deg(v) + 2^i$

m_i

$m^{(\ell+1)} \leq \underset{\sim}{\Delta n}$

$2\Delta n$

$2^i \cdot n$

$\|$

Δ

leftover $\frac{2\Delta n}{4}$

$\frac{\Delta n}{2}$